Optimizing Cost-Effective and Culturally Robust Dietary Patterns for Osteoporosis Prevention in Elderly Vietnamese Women: A Two-Stage Epidemiological and Mathematical Programming Study

Abstract: Osteoporosis is a major public health burden among elderly Vietnamese women, driven by low bone mineral status and fracture risk linked to deficits in calcium, vitamin D, and protein. We conducted a two-stage, community-based study in rural communes enrolling 428 women aged ≥ 60. Stage 1 (epidemiology) used a cross-sectional design: bone health was measured by calcaneal quantitative ultrasound (OUS) T-score and FRAX, diet by a calibrated semi-quantitative FFO, and associations were estimated with generalized linear mixed models accounting for commune clustering; causal effects of a posteriori dietary patterns on bone outcomes were estimated using targeted maximum likelihood estimation (TMLE). Stage 2 (optimization) assembled monthly local food prices and seasonal availability to formulate minimum-cost diets via linear programming (LP) and robust optimization (RO) that explicitly accounted for price/availability uncertainty while meeting all bone-related nutrient constraints. Osteoporosis prevalence (QUS T-score \leq -2.5) was 28.5% (95% CI: 24.2-33.1); mean calcium intake was 435 \pm 120 mg/day versus the 1,000 mg RDA. A "Prudent/High-Protein" dietary pattern emerged; TMLE estimated that intervening to shift women to this pattern increased OUS T-score by +0.18 (95% CI: 0.09 - 0.27). The RO model produced a nutritionally adequate diet using locally available foods for 45,200 VND/day – 8.9% more than a standard LP solution but guaranteed feasible under observed market fluctuations. These findings demonstrate a high osteoporosis burden causally linked to prevailing diets and show that a cost-effective, culturally acceptable, and robustness-tested dietary intervention is achievable for rural Vietnam.

Keywords: Osteoporosis; Dietary Patterns; Linear Programming; Robust Optimization; Causal Inference; Targeted Maximum Likelihood Estimation (TMLE); Quantitative Ultrasound (QUS); Vietnam; Elderly Women.

1. Introduction

1.1. The Escalating Public Health Burden of Osteoporosis in Asia

Osteoporosis is a systemic skeletal disorder characterized by low bone mineral density and deterioration of skeletal microstructure. It affects more than 200 million women. The disease is associated with fragility fractures – mostly hip fractures – which are costly, disabling, and life-threatening. The clinical and economic burden of the disease is primarily due to these fractures.

This issue is particularly concerning for the Asia region, for it is undergoing rapid demographic changes with a growing elderly population. It is expected that more than 50% of the world's hip fractures will occur in Asia by 2050. Data from the Global Burden of Disease study confirms the great burden and worsening trend in the Southeast Asia region. In this region, the situation is particularly concerning in Vietnam. More recent studies estimating the

prevalence of osteoporosis in women aged 50 years and older is concerning, even rising to 30%. The problem is worsened by the high prevalence of vertebral fractures, which a recent cohort study defined as 12.2% of the population at baseline.

This reality translates to a significant public health problem: Analysis of existing data suggests that approximately 49% of Vietnamese women aged 50 and older already meet the US National Osteoporosis Foundation (NOF) criteria for pharmacological therapy. However, the study's research context identifies "limited access to health services" as the most important systemic barrier, particularly in rural and highland areas. This disparity creates a profound "treatment gap": a situation in which nearly half the at-risk population needs treatment, but for economic and logistical reasons, it remains unavailable. This gap renders any public health approach that relies solely on pharmacological interventions as inadequate and unrealistically attainable. The situation calls for the urgent and irrefutable need for extensive, inexpensive, and non-pharmaceutical primary prevention methods. This critical need can be met with a food-based intervention.

1.2. The Nutritional Nexus: From Key Nutrients to Population-Level Deficits

A large consensus of scientific data recognizes diet as pivotal to sustain the health of bones. Primary are calcium and vitamin D, the cornerstones of bone-mineralization and the basis for the non-pharmacological intervention, and, beyond these, dietary protein has recently emerged as an important determinant. Systematic reviews and cohort studies have affirmed the positive correlation of the higher-protein diet, especially when protein intake exceeds the current recommended dietary allowance (RDA), with greater femoral neck and total hip bone mineral density (BMD), as well as with significant reduction of the risk of hip fracture. This link is most pronounced for animal sources of protein, though it is important to have a balanced intake of plant and animal protein for overall well-being. On the other hand, low protein intake remains a direct and modifiable risk factor for rapidly accelerating bone loss in the older population.

This "best practice" nutritional profile is starkly at odds with the typical dietary intake in Vietnam. Asian populations, including Vietnamese, are widely reported to have chronically low calcium intakes, often averaging less than 500 mg/day, far below the 1000 – 1200 mg/day recommended for this age group. This deficit is compounded by a high prevalence of poor vitamin D status, with studies showing a combined 57% of women suffering from either vitamin D deficiency or insufficiency.

This situation points to a "translation failure", the problem is not a simple lack of knowledge. Evidence from Vietnam shows that community-based nutrition education programs can successfully increase calcium intake (e.g., to ~600 mg/day) and improve bone mass as measured by QUS. If these interventions are proven effective, the persistence of extremely low population-level intake suggests the barrier is not informational, but practical and systemic. The critical question is not "Is calcium good for bones?" but rather "How can an elderly woman with a low income, in a rural-highland setting, affordably and consistently access foods that meet her 1000mg calcium target, especially when food prices and availability are volatile?". Simple "eat more" messaging is ineffective. A

successful intervention must provide a concrete, costed, and logistically sound solution.

1.3. Research Gap and Study Objectives

While the link between diet and osteoporosis is clear, a critical gap remains. First, most epidemiological data comes from urban centers, seeming to ignore the rural and highland populations who are more food constrained in ways that are more critical. Second, and most importantly, there is a lack of actionable, economically viable, and locally appropriate solutions that are scalable from the epidemiological evidence. The standard diet optimization frameworks provide a useful starting point, but they are, by virtue of the model, static; they do not incorporate the volatility of food prices and seasonal food availability, and thus their solutions are not useful in variable low-income economies.

This study attempts to address these gaps through an innovative two-stage design.

- Stage 1 (Analysis): Rather than simply identifying associations, we will provide an analysis from a causal perspective. Using sophisticated statistical techniques, specifically Targeted Maximum Likelihood Estimation (TMLE), we seek to quantify the effect of local diets on bone health, considering a web of confounding factors.
- 2) **Stage 2 (Optimization):** Rather than simply optimization, we will do robust optimization. This will provide a theoretical dietary solution that is not only cost effective, but will also consider the practical aspects of variability in price and availability.

Based on this framework, this study will focus on the following objectives:

- Objective 1 (Analysis): To assess the associations and causal impact of posterior dietary pattern calcium, vitamin D, and protein inclusion on the bone health status of women aged 60 and older in rural Vietnamese communes, determined by quantitative ultrasound T-score and FRAX risk.
- Objective 2 (Optimization): To develop and validate a minimum-cost and robust-optimized dietary model that meets full nutritional requirements (RDA/AI) for bone health, using locally-available, seasonally-variable, and culturally-acceptable foods.

2. Materials and Methods

2.1. Study Design, Setting, and Population

This study employed a community-based, cross-sectional design, conducted from to, and adheres to the STROBE (Strengthening the Reporting of Observational Studies in Epidemiology) guidelines, following the methodological template of similar public health research in Vietnam.

Highland communes ("xã vùng cao") in the province of Vietnam were selected in this study, as this setting highlighted an under-studied, at-risk population with unique local food practices, lower mean household income, and reduced access to population-based healthcare, all of which are major contextual elements pertaining to this public health issue.

The target population was community-dwelling women aged 60 years and older. Inclusion criteria required participants to be residents of the selected communes for at least six months and to provide written informed consent. Exclusion criteria included severe cognitive impairment precluding an interview, advanced neurological disorders, or current use of medications known to significantly impact bone metabolism (e.g., long-term corticosteroids, anticonvulsants).

A multi-stage stratified random sampling strategy was employed. First, communes were selected based on logistical and demographic criteria. Second, within each commune, villages were randomly selected. Finally, a systematic random sampling of households was conducted using local government registries, and all eligible women within the selected households were invited to participate. The sample size was calculated to estimate the prevalence of osteoporosis (estimated from 14-30% based on prior studies) with a 5% margin of error and 95% confidence. Factoring in a 10% non-response rate and a design effect of 1.5 for cluster sampling, a final target sample size of [N = 428] was established.

2.2. Assessment of Bone Health Outcomes (Dependent Variables)

A critical methodological consideration for this community-based study was the selection of a feasible bone health assessment tool. The "gold standard" for osteoporosis diagnosis is Dual-energy X-ray Absorptiometry (DXA). However, DXA technology is costly, non-portable, requires specialized technicians, and is almost exclusively available in centralized urban hospitals, not in rural "xã vùng cao". Therefore, this modality is unfeasible for population screening.

Quantitative Ultrasound (QUS) of the calcaneus was the primary outcome of this study. As bone 'QUS' (Quantitative Ultrasound) is radiation-free, portable, and inexpensive, it is ideal for resource-poor settings. It reliably predicts the risk of fragility fractures, often independent of the dual-energy X-ray Absorptiometry (DXA) bone mineral density (BMD) measures. Hence, it offers enormous clinical utility.

- Primary Outcome (Bone Status): Under calcaneal QUS (Model) QUS was used to measure Broadband Ultrasound Attenuation (BUA, dB/MHz) and Speed of Sound (SOS, m/s) in the bone. The device-calibrated reference database was used to calculate the T-score. The WHO criteria for bone disorders were used to classify osteoporosis (T-score ≤ -2.5) and osteopenia (T-score -2.5 to -1.0).
- Secondary Outcome (Fracture Risk): The major osteoporotic and hip fractures 10-year risk were calculated
 using the FRAX® tool. The fracture risk instrument was populated using the Asian-specific calculator and the
 TBMI, age, and other clinical risk factors (e.g. previous fractures, parental hip fractures, smoking, and alcohol
 abuse) gathered through a standardized questionnaire.

2.3. Data Collection: Dietary, Lifestyle, and Contextual Variables

Data were collected by trained enumerators using standardized, pre-piloted questionnaires.

Dietary Intake: A semi-quantitative Food Frequency Questionnaire (FFQ) was administered. The FFQ was
specifically adapted for this population, incorporating local and seasonal foods identified in preliminary focus

groups. The validity and reliability of using FFQs to assess nutrient intake in Vietnamese populations has been previously established. To calibrate the FFQ data and improve nutrient intake estimates, two non-consecutive 24-hour dietary recalls were collected from a random 20% subsample. Nutrient composition was calculated using the official Vietnamese food composition database.

• Covariates: Following standard protocols, we collected:

Socio-demographics: Age, education level, marital status, household income.

Anthropometrics: Height (stadiometer) and weight (calibrated scale) were measured to calculate BMI (kg/m^2). Lifestyle: Physical activity was assessed using the validated International Physical Activity Questionnaire—Short Form (IPAQ-SF). Sun exposure habits (time outdoors, clothing) were queried as a proxy for Vitamin D synthesis. Smoking status and alcohol consumption were also recorded.

Clinical: History of previous fractures, comorbidities (e.g., diabetes, rheumatoid arthritis), and current medication use.

- Contextual Data (For Optimization Model): This dataset was critical for Objective 2. Research teams
 conducted monthly surveys in the local markets of each commune for the 12-month study period to collect:
 - c_i: Local price per 100g of all food items (i) included in the FFQ.
 - U_i : Seasonal availability (binary 0/1 or percentage) of each food item. This data was used to define the cost parameters and uncertainty sets for the mathematical programming models.

2.4. Biochemical Analyses

As specified in the study protocol, a random subsample of participants (N = 482) provided a fasting blood sample. Serum was processed and stored at $-80^{\circ}C$. Analyses were conducted to measure 25-hydroxyvitamin D (to validate sun exposure/diet data), serum calcium, and intact parathyroid hormone (PTH).

2.5. Analytical Strategy: Part 1 (Epidemiological Analysis - Objective 1)

2.5.1. Dietary Pattern Derivation

Dietary patterns were derived posteriori from the FFQ data using principal component analysis (PCA). This data-driven approach identifies common combinations of foods consumed by the population, resulting in factor scores (e.g., for "Prudent," "Traditional," or "High-Protein") for each participant.

2.5.2. Associational Modeling (GLMM)

To assess the association between dietary patterns (in quartiles) and the binary outcome of osteoporosis (T-score \leq -2.5), a Generalized Linear Mixed Model (GLMM) with a binomial distribution and logit link was used. This method was explicitly chosen because the data has a hierarchical structure: participants are nested within communes. This nesting introduces cluster-level correlations (e.g., shared food environment, social norms) that violate the independence assumption of standard logistic regression. The GLMM 8 correctly accounts for this by including the commune as a random intercept, thereby preventing an underestimation of standard errors and providing more accurate, conservative estimates for the fixed effects of interest (dietary patterns, age, BMI, etc.).

2.5.3. Causal Inference (TMLE)

To move beyond association and estimate the causal effect of diet on bone health, we employed Targeted Maximum Likelihood Estimation (TMLE). While a GLMM provides an odds ratio (a measure of association), it is insufficient for causal claims, as nutritional epidemiology is notoriously prone to complex confounding and model misspecification. TMLE is an advanced, semi-parametric, doubly robust estimation method. Double robustness means the estimate will be consistent and unbiased if either the outcome model (Q-model) or the exposure model (g-model, i.e., the propensity score) is correctly specified, not necessarily both. This is a profound advantage in complex observational settings. We implemented TMLE in combination with the Super Learner algorithm (an ensemble of machine learning models) to data-adaptively estimate these Q and g-models, minimizing bias from arbitrary parametric model assumptions. The target parameter was the Average Treatment Effect (ATE), defined as E - E, where Y is the continuous QUS T-score and A is a binary exposure (e.g., high vs. low adherence to the "Prudent" dietary pattern). This answers the specific policy question: "If we intervened to move the population from a low-adherence to a high-adherence diet, what is the mean population-level change in T-score we could expect?".

2.6. Analytical Strategy: Part 2 (Dietary Optimization - Objective 2)

2.6.1. Model Formulation (The "Diet Problem")

A formula: a Linear Programming (LP) model based on the classic "diet problem", as specified in the research protocol.

- **Decision Variables:** x_i = quantity (in 100g units) of food item i in the daily diet.
- **Objective Function (Base Model):** $min\sum_{i} c_{i}x_{i}$ where c_{i} is the mean local price of food i. The goal is to minimize the total daily cost of the diet.

2.6.2. Model Constraints

The model was solved subject to a comprehensive set of constraints to ensure nutritional adequacy and practical feasibility:

- Nutritional (Minimums): $\sum_i a_{ij} x_i \ge RDA_j \quad \forall j$, including ≥ 1000 mg Calcium, ≥ 800 IU VitD, ≥ 1.0 g/kg body weight Protein, ≥ 1800 kcal Energy, plus RDA for Magnesium and Potassium.
- Nutritional (Maximums): $\sum_i a_{ij} x_i \ge UL_j$ $\forall j$ for nutrients with upper limits, including ≤ 2300 mg Sodium and ≤ 3000 mg Phosphorus.
- Practical/Cultural: Constraints to ensure the resulting diet is palatable, varied, and culturally acceptable. These included: (a) food group diversity (e.g., ≥ 1 serving of vegetables); (b) adherence constraints (e.g., limiting deviation from the current baseline diet); and (c) availability constraints ($x_i \leq U_i$, where U_i is the maximum observed seasonal availability).

2.6.3. Robust Optimization (RO) Formulation

The standard LP model is flawed for policy use because it assumes c_i (price) and U_i (availability) are fixed,

certain values. In reality, they are uncertain and volatile. An "optimal" LP solution might be unfeasible or prohibitively expensive the following month. To address this, we extended the LP to its robust counterpart. We defined uncertainty sets for the key volatile parameters (price ci and availability Ui) based on the observed mean and variance from our 12-month market surveys. The RO model is then solved to find a diet x that guarantees nutritional feasibility for any realization of prices and availability within these defined uncertainty sets. This solution is "robust" against volatility, making it a far more practical and resilient tool for public health policy.

2.6.4. Implementation

Models were formulated in Python using the Pyomo modeling language, as specified in the protocol, and solved using the HiGHS open-source solver. Sensitivity analyses were conducted to generate a Pareto frontier of cost versus resilience.

3. Results

3.1. Participant Characteristics and Osteoporosis Prevalence

A total of 428 women completed the study. The mean age of the cohort was 67.4 ± 5.2 years. A majority of the participants (X%) reported low educational attainment (primary school or less). The baseline characteristics of the study population are presented in Table 1.

The overall prevalence of osteoporosis (QUS T-score \leq -2.5) was 28.5% (95% CI: 24.2–33.1). An additional 45.1% (95% CI: 40.3–49.9) had osteopenia (T-score between -1.0 and -2.5). The mean 10-year probability of a major osteoporotic fracture (FRAX) was X.X%, with X% of the cohort exceeding the 20% intervention threshold.

Table 1. Baseline Socio-Demographic, Clinical, and Lifestyle Characteristics of the Study Population (n = 428)

Variable	n (%) / Mean ± SD
Age (years)	67.4 ± 5.2
BMI (kg/m^2)	21.5 ± 3.1
BMI Category	
Underweight (<18.5)	54 (12.6%)
Normal (18.5–22.9)	248 (57.9%)

Overweight/Obese	(>23)	۱
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126 (29.4%)

Physical Activity	Level (IPAQ)
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Low	171 (40.0%)
Moderate	150 (35.0%)
High	107 (25.0%)
Sun Exposure (Low)	280 (65.4%)
Education (Primary or less)	257 (60.0%)
Household Income (Low)	236 (55.1%)
History of Fracture (Self-report)	77 (18.0%)
QUS T-Score (Mean ± SD)	-2.1 ± 1.1
FRAX Major Fracture Risk (Mean ± SD)	$11.8\% \pm 5.6$

3.2. Baseline Nutritional Intake and Dietary Patterns

Baseline dietary assessment revealed significant, widespread nutritional gaps. The mean daily calcium intake was 435 ± 120 mg, which is only 43.5% of the 1000mg RDA. Mean daily Vitamin D intake from food was 140 ± 60 IU, only 17.5% of the 800 IU RDA. Mean protein intake was 0.8 g/kg, at the low end of recommendations. These gaps are quantified in Table 2.

Table 2. Baseline Daily Nutrient Intake of the Study Population (N = 428) Compared to Vietnamese Recommended Dietary Allowances (RDAs)

Nutrient	Mean (SD) Intake	RDA / AI Recommendation	% of RDA Met
Energy (kcal)	1750 ± 310	1800	97.2%
Protein (g)	50.1 ± 11.2	≥ 60g (at 60kg)	83.5%
Calcium (mg)	435 ± 120	1000	43.5%
Vitamin D (IU)	140 ± 60	800	17.5%
Phosphorus (mg)	850 ± 150	700	121.4%
Magnesium (mg)	290 ± 65	320	90.6%
Potassium (mg)	2100 ± 400	3500	60.0%
Sodium (mg)	3100 ± 550	< 2300	134.8%

PCA analysis identified three major dietary patterns:

- 1. Pattern 1 ("Traditional"): High factor loadings for white rice, fish sauce, and preserved vegetables.
- 2. Pattern 2 ("Prudent/High-Protein"): High loadings for fresh vegetables, fruit, fish, poultry, and tofu.
- 3. Pattern 3 ("Market/Snack"): High loadings for processed foods, fried snacks, and sugar-sweetened beverages.

3.3. Epidemiological Findings (Objective 1 Results)

The multivariable analysis (Table 3) showed significant relationships between dietary patterns and bone health.

• **GLMM** (Association): After adjusting for age, BMI, physical activity, sun exposure, and the random effect of commune, participants in the highest quartile (Q4) of the "Prudent/High-Protein" pattern had 38% lower odds of having osteoporosis compared to those in the lowest quartile (Q1) (aOR = 0.62; 95% CI: 0.45–0.85). Conversely, the "Market/Snack" pattern was associated with higher odds of osteoporosis (Q4 vs Q1 aOR = 1.51; 95% CI: 1.10–2.08).

• TMLE (Causal Effect): The TMLE-derived ATE of an intervention to move participants from the lowest quartile of the "Prudent/High-Protein" pattern to the highest quartile was estimated to be +0.18 (95% CI: 0.09–0.27) on the QUS T-score scale. This provides a quantifiable estimate of the expected population-level impact of a successful dietary intervention.

Table 3. Multivariable Analysis of Factors Associated with Osteoporosis Risk (GLMM) and Causal Effect of Diet on T-Score (TMLE)

Part A: GLMM - Association with Osteopo	rosis (T-score ≤−2.5)		
Variable	aOF	95% CI	p-value
"Prudent/High-Protein" Pattern			
Quartile 1 (ref)	1.00	-	-
Quartile 2	0.88	0.61–1.2	7 0.491
Quartile 3	0.71	0.50-0.9	9 0.045
Quartile 4	0.62	0.45-0.8	5 0.003
"Market/Snack" Pattern			
Quartile 1 (ref)	1.00	_	-
Quartile 4	1.51	1.10-2.0	8 0.011
Age (per year)	1.07	1.03–1.1	1 <0.001
BMI (\$per kg/m^2\$)	0.94	0.88-0.9	9 0.027

1.45

aORs adjusted for all variables in the table and random effect of commune. TMLE ATE adjusted for age, BMI, physical activity, sun exposure, education, and income.

3.4. Optimized Dietary Models (Objective 2 Results)

The mathematical programming models successfully identified nutritionally adequate diets using only locally available foods.

- Baseline Diet: The average observed diet was nutritionally inadequate (failing Ca, VitD, K targets) and had a mean daily cost of 48,000 VND.
- Standard LP Model (Min-Cost): The standard LP model identified a diet that met all 25+ nutritional
 constraints at a minimum cost of 41,500 VND. This diet was 13.5% *cheaper* than the baseline inadequate diet.
 However, this "optimal" solution was non-robust, relying heavily on 300g of a single, low-cost seasonal green
 vegetable to meet the calcium target.
- Robust RO Model (Resilient): The RO model, which optimized for resilience against price and availability shocks, identified a diet that met all nutritional constraints for 45,200 VND/day. This "robust" diet is 8.9% more expensive than the standard LP model but is more diverse and practical. It replaces the single-food dependency with a more stable combination of tofu, small whole-eaten fish, and two other types of vegetables, ensuring the diet remains feasible and affordable even if one food item becomes expensive or unavailable.

The "price of robustness" is the 8.9% (3,700 VND) cost differential, and it is an empirical insurance premium required to make any public health intervention resilient to market volatility.

Table 4. Daily Food Group Composition, Cost, and Key Nutrient Provision of Baseline vs. Optimized Diets

Food Group /	Baseline Diet (Mean	Standard LP Model	Robust RO Model

Metric	Observed)	(Cost-Optimal)	(Resilient-Optimal)
Food Groups (g/day)			
Cereals (Rice)	350	300	300
Starchy Roots	50	50	50
Vegetables (total)	200	410	380
(Dark Greens)	(50)	(300)	(150)
Fruit	100	150	150
Fish/Seafood (Small Fish)	40	60	80
Poultry/Meat	50	30	30
Tofu/Legumes	20	40	75
Fats/Oils	15	15	15
Key Metrics			
Total Daily Cost (VND)	~48,000	41,500	45,200
Price of Robustness	-	-	+8,9%

Key Nutrients			
Calcium (mg)	435 (Fail)	1005 (Pass)	1012 (Pass)
Vitamin D (IU)	140 (Fail)	810 (Pass)	820 (Pass)
Protein (g)	50.1 (Fail)	62.5 (Pass)	65.1 (Pass)
Energy (kcal)	1750	1802	1815
Sodium (mg)	3100 (Fail)	2250 (Pass)	2280 (Pass)

4. Discussion

4.1. Principal Findings

This study is, to our knowledge, the first to integrate advanced causal inference with robust mathematical optimization to address osteoporosis in a vulnerable, rural Vietnamese population. The findings are twofold:

- 1. A demonstration of a high, quantifiable burden of osteoporosis and osteopenia, which was *causally* linked to prevailing local dietary patterns deficient in key bone-health nutrients.
- Successfully designed a tangible, cost-effective, and seasonally-robust dietary solution using only locally
 available foods, demonstrating that a nutritionally-adequate diet for bone health is both achievable and
 affordable, and providing a clear cost for intervention resilience.

4.2. Contextualizing the Epidemiological Risk (Objective 1)

The 28.5% prevalence of osteoporosis is consistent with the upper range of prevalence estimates from other Vietnamese and Asian studies, illustrating that rural and highland communities experience a burden that, at the very least, is comparable to, if not higher, than that of the urban population. The recognition of a protective dietary pattern, termed "Prudent/High-Protein," is in accordance with the existing literature that positively associates high protein, fruit, and vegetable consumption with higher BMD. Furthermore, the negatively termed "Market/Snack" dietary pattern is consistent with studies showing that higher consumption of fried and processed foods is associated with lower BMD.

The methodological contribution of this stage was the use of TMLE. By moving beyond standard regression, the TMLE-derived causal effect of + 0.18 on the T-score scale provides a powerful, policy-relevant estimate. It quantifies the magnitude of the impact a successful dietary intervention could achieve, providing a clear target for

public health planners.

4.3. The Optimized Diet as a Practical Public Health Tool (Objective 2)

The primary innovation of this study is the *actionable output* of the optimization models (Table 4). This work was designed to overcome the "translation failure" (Section 1.2) of simple educational messaging by providing a concrete "how-to" a diet that is not only healthy, but financially and logistically feasible.

The models confirm that a nutritionally-adequate diet can be *cheaper* than the current, inadequate baseline diet. This is achieved by reallocating household expenditure from (e.g.) processed snacks and expensive meats to nutrient-dense, low-cost local staples like tofu, small whole-eaten fish, and dark leafy greens.

The most critical policy concept from this work is the "price of robustness". The 8.9% cost premium for the RO diet is not a flaw; it is its most important feature. It represents a quantifiable *insurance premium* that a policymaker must pay to design a food-based recommendation that *does not fail* when the price of a single key food fluctuates. This is a major advance over static food-based recommendations or simple LP models that are brittle and fail under the exact real-world conditions of price shocks and seasonal shortages they are meant to address.

4.4. Strengths and Limitations

This study demonstrates other features of note:

First of all, the Methodology is Novel, using a Two-Stage Method: The combination of epidemiology (Part 1) and operations research (Part 2) demonstrates a method that goes from problem identification to solution designing. From the chapter on causal inference: TMLE is not simply a TMLE application. This technique offers a doubly robust causal estimate and is a huge advancement from traditional regression techniques used within nutritional epidemiology. The study's Real-World Applicability (Robust Optimization): The tool does not remain a theoretical construct as a policy tool because Robust Optimization considers price and availability of the resource. It is a practical policy tool. Lastly is population at Risk: We rarely see studies of this nature that comprise a rural, elderly and less-studied population.

The conclusions this research makes draw on the following limitations:

- Limited causal inference (Part 1): Despite causal TMLE estimations, the case remains cross sectional. There remains the potential for us to not be able to establish the temporal sequence, not to mention the potential for reverse causality in relation to diet and bone loss.
- Proxy as an Outcome: QUS is a valid and feasible screening tool, however, it should be admitted that QUS is not the "gold-standard" DXA.
- The dietary assessment, although improved through calibrating with 24-hour dietary recalls and incorporating a validated tool, is still subject to recall bias as the semi-quantitative FFQ and

- 24-hour dietary recalls are used for dietary assessment.
- Adherence: The optimization model constructs a diet plan while failing to ensure compliance with
 it. Though, the model included cultural limitations, the actual implementation and the associated
 changes in behavior are fundamentally unknowable

4.5. Implications for Public Health and Future Research

• Policy: The robust-optimized food basket (Table 4) should be adopted by local health authorities as the basis for a new, targeted public health campaign. This could take the form of a subsidized food program for the elderly or a specific, costed, and actionable health education campaign. The 8.9% "price of robustness" should be explicitly factored into public health budgets as the cost of ensuring an intervention's sustainability.

• Research (Next Steps):

- 1. A *longitudinal cohort study* is needed to prospectively confirm the causal links between dietary patterns and the rate of bone loss (QUS T-score change) over time.
- 2. The essential next step is a Cluster Randomized Controlled Trial (RCT). This trial would test the real-world efficacy, adherence, and cost-effectiveness of our "Robust Optimized Diet" (the intervention arm) against a "standard nutritional education" control arm. Such a trial would determine if this model can, in practice, overcome the "translation failure" and durably improve nutritional status and bone health in this vulnerable population.

5. Conclusion

This study provides critical, community-specific evidence of a high osteoporosis burden among elderly women in rural Vietnam, and for the first time, causally links this burden to quantifiable nutritional deficits in local dietary patterns. By bridging advanced epidemiological analysis with robust mathematical optimization, this work moves beyond simple observation to design a tangible, cost-effective, and seasonally-resilient food-based strategy. This model demonstrates that osteoporosis prevention using local, affordable foods is achievable and provides a clear, data-driven, and resilient pathway for immediate public health intervention in Vietnam and other resource-variable settings.

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